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Transfinite Interpolation of Steam Tables

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In engineering practice, real time simulation, reactor accident analysis, etc. it is often necessary to obtain very accurate but simple relationships between steam and water properties with minimum computer storage requirements. The notion of "transfinite element," developed by Gordon and Hall, is used as a basis for an interpolation scheme which gives simple functional relationships between steam and water properties, such as enthalpy, considered as a function of pressure and temperature.

1. INTRODUCTION

In the design and operation of nuclear and fossil fired power plants accurate interpolation of tabulated thermodynamic properties of steam and water are necessary. Since small or medium sized computers are involved, and very often in a real time environment, it is desirable to have the capability of retrieving good approximate values of these thermodynamic properties while only having to store a minimal amount of data. In general, steam and water properties can be considered as a surface in temperature–pressure–enthalpy space or in temperature–pressure-specific volume space, etc., and their approximation reduces to a surface fitting problem not totally unlike those considered in the styling of automobile exteriors [3, 4, 5].

Recently, there has been great interest in developing automatic mesh generators for use in the finite element method. The purpose of this paper is to indicate the application of these techniques, especially the notion of transfinite elements [8, 9], to the approximation and interpolation of steam and water properties. The

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emphasis is on the storage and computational problems encountered when small or medium size computers are to be used, as is the case in the application of power plant process computers.

2. Background Information

In the design of nuclear and fossil fired power plants the use of the thermodynamic properties of steam and water is essential. These properties are published as a set of tables relating temperature, pressure, enthalpy, specific volume and entropy. Their nature is quite disjoined, since water and steam exhibit unrelated behavior at different temperatures and pressures. In most cases, the steam tables, as they are popularly known, cover the range from the freezing point (32.018°F) to the critical point (705.47°F) on the saturation line, and from 0 to 15,000 psia and 32°F to 1500°F in the compressed liquid or superheated steam region.

This is of course a very broad range of temperatures and pressures and computationally efficient approximation is a large scale problem. Since the development of process control computers the necessity of the accurate but fast calculation of these properties became widespread. In many instances, high degree polynominals (sixth or seventh degree polynominals using natural logarithms and exponents) were used to approximate thermodynamic properties in adjacent regions [1, 2]. In large computers, used for design purposes, very general subroutines have been written which use tensor product bilinear [1] and biquadratic [12] interpolation between the points. In many such cases virtually complete tables are stored in the computer memory, yielding fairly accurate but very inefficient computer subroutines. In process control computers working in real time environment, or in real time simulation, such an approach seems to be not only undesirable but also impossible to achieve. One should note, that the size of a process control computer is approximately 25 to 30 k of core while such a steam table approximation would take approximately 25 k of core by itself. In addition to the storage problem, the computational time is of great importance. In an average size nuclear power plant simulation there are up to one half million calls of the steam table routines per one time step. Therefore, a compromise should be found between the accuracy, speed of the calculation and computer memory requirements.

It was reported earlier [6] that univariate cubic splines were used successfully in the approximation of the saturation properties in the range from 70 to 700°F with the use of an automatic node search program. In this application six or seven nodes were sufficient to achieve the accuracies of about 0.1 %. However, when the compressed liquid and superheated steam properties are needed, *surface* approximation is necessary. Due to the disjoined nature of the properties mentioned earlier, piecewise bivariate polynominals are ideally suited for this task. There

were experiments with the use of bilinear and bicubic splines as well as the combination of cubic splines and linear interpolation [7]. However the authors feel that the use of blending function techniques [4, 5] which fostered the notion of a transfinite element, lead to more efficient, flexible and accurate schemes using a relatively small amount of data storage. For the convenience of the reader, some results developed in [8, 9] will be repeated here.

3. Blending Function Techniques

Transfinite interpolation is by definition an interpolation method in which the interpolant matches a given function on a nondenumerable or "transfinite"

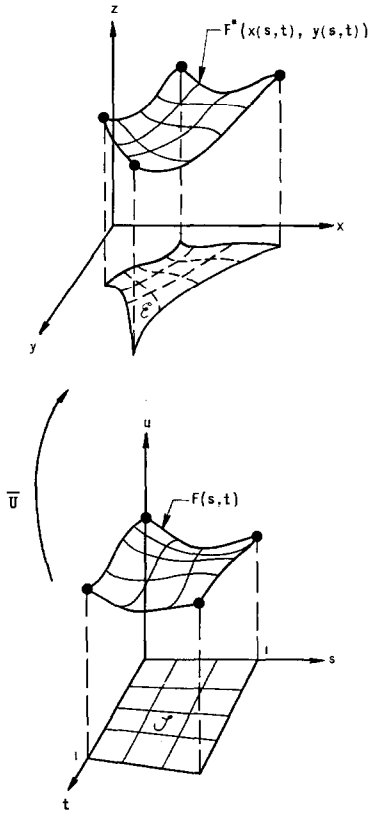

FIG. 1. A univalent mapping $\mathcal{O}$ induces a curvilinear coordinate system on the domain $\mathcal{E}$ and the interpolation–approximation problem can be described and then solved in terms of the s, t system.

set of points. This is in contrast to the "classical" multidimensional interpolation schemes which match (or interpolate to) the given function on a denumerable or finite set of points (cf. [9]).

Ultimately, we seek interpolation schemes over curved domains, however as in [9] we first consider a square domain $\mathscr{S}$ as in Fig. 1. Assume the curves $\{F(s_i, t)\}_{i=0}^{m}$ and $\{F(s, t_j)\}_{j=0}^{n}$ are known or have been approximated to a high degree of accuracy (here $m = 3$ and $n = 4$). Define the projectors $\mathscr{P}_s$ and $\mathscr{P}_t$ by

$$\mathscr{P}_s[F] = \sum_{i=0}^{m} F(s_i, t)\, \phi_i(s) \tag{1}$$

$$\mathscr{P}_t[F] = \sum_{j=0}^{n} F(s, t_j)\, \psi_j(t) \tag{2}$$

where, for example,

$$\phi_i(s) = \frac{\prod_{k \neq i} (s - s_k)}{\prod_{k \neq i} (s_i - s_k)}, \qquad \psi_j(t) = \frac{\prod_{k \neq j} (t - t_k)}{\prod_{k \neq j} (t_j - t_k)}. \tag{3}$$

We note, that $\mathscr{P}_s[F]$ interpolates to $F(s_i, t)$, $0 \leqslant i \leqslant m$, $0 \leqslant t \leqslant 1$ and $\mathscr{P}_t[F]$ interpolates to $F(s, t_j)$, $0 \leqslant j \leqslant n$, $0 \leqslant s \leqslant 1$. That is, $\mathscr{P}_s[F] = F$ and $\mathscr{P}_t[F] = F$ for a transfinite (more than finite) number of points.

The functions $\{\phi_i(s)\}$ and $\{\psi_j(t)\}$ are termed *blending functions* [4] and need only satisfy the cardinality conditions

$$\phi_i(s_k) = \delta_{ik}, \qquad \psi_j(t_k) = \delta_{jk}, \tag{4}$$

where δ is the Kronecker delta. Other choices for the ϕ_i and ψ_j can be made [4, 5, 9, 10].

For bivariate interpolation, the bipolynomial or *tensor product* Lagrange interpolation formula can be used and is given by

$$\mathscr{P}_s\mathscr{P}_t[F] \equiv \mathscr{P}_s[\mathscr{P}_t[F]] = \sum_{i=0}^{m} \sum_{j=0}^{n} F(s_i, t_j)\, \phi_i(s)\, \psi_j(t). \tag{5}$$

Note that $\mathscr{P}_s\mathscr{P}_t[F]$ interpolates to F at the $(m + 1) \times (n + 1)$ points (s_i, t_j), $0 \leqslant i \leqslant m$, $0 \leqslant j \leqslant n$. In this case the precision set of the product projector is finite while each projector, per se, has transfinite precision set.

The Boolean sum of the projectors [4, 5]

$$\mathscr{P}_s \oplus \mathscr{P}_t[F] = \mathscr{P}_s[F] + \mathscr{P}_t[F] - \mathscr{P}_s\mathscr{P}_t[F] \tag{6}$$

is also a projector and interpolates to F along the $(m + n + 2)$ lines $s = s_i$,

$0 \leqslant i \leqslant m$ and $t = t_j$, $0 \leqslant j \leqslant n$. $\mathscr{P}_s \oplus \mathscr{P}_t[F]$ is called the *transfinite bivariate Lagrange interpolant* [9] or in earlier terminology *blended interpolant* [4, 5].

By proper choice of m, n, ϕ, ψ one obtains various transfinite interpolants. For sufficiently smooth functions error bounds have been derived for such schemes [8, 9, 10] and in particular for the Lagrange blending functions (3) we have [9], with $\| g \|_\infty \equiv \max_{(s,t)\in\mathscr{S}} | g(s, t)|$, that

$$\|(F - \mathscr{P}_s \oplus \mathscr{P}_t[F])^{(k,l)} \|_\infty \leqslant \epsilon_{mk}\epsilon_{nl} \| F^{(m+1,n+1)} \|_\infty h^{m+n+2-k-l} \tag{7}$$

for $F \in C^{(m+1,n+1)}(\mathscr{S})$, $0 \leqslant k \leqslant m$, $0 \leqslant l \leqslant n$. For bilinear transfinite interpolation and $F \in C^{(2,2)}$ this error bound yields

$$\| F - \mathscr{P}_s \oplus \mathscr{P}_t[F]\|_\infty \leqslant (\epsilon_{10})^2 \| F^{(2,2)} \|_\infty h^4 \tag{8}$$

and for the biquadratic transfinite interpolation for $F \in C^{(3,3)}$

$$\| F - \mathscr{P}_s \oplus \mathscr{P}_t[F]\|_\infty \leqslant (\epsilon_{20})^2 \| F^{(3,3)} \|_\infty h^6. \tag{9}$$

Note, that in contrast, the bilinear tensor product projection would match the given function only at four points (four corners), the biquadratic tensor product projection at only nine points and the associated error bounds are $O(h^2)$ and $O(h^3)$, respectively.

Since in engineering applications we normally deal with functions whose values are known only at a finite number of points, we have to approximate the univariate functions $F(s_i, t)$ and $F(s, t_j)$ by some approximation scheme. Because of the

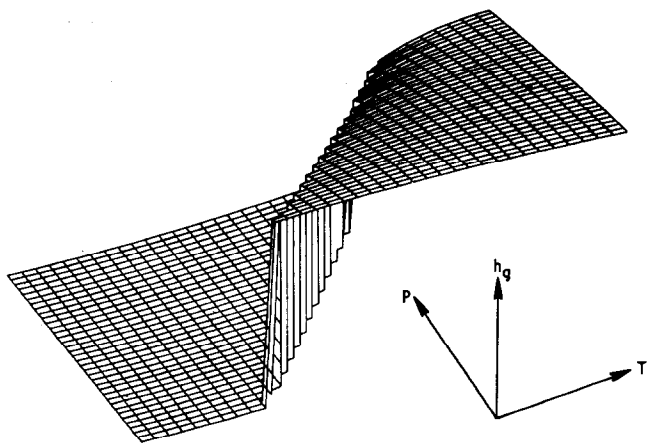

FIG. 2. Enthalpy of superheated steam versus temperature and pressure (Narrow range).

disjoined nature of the steam and water properties, the use of piecewise polynominals, linear and cubic splines, appear to be the natural choices in this approximation.

Interpolation of steam and water properties in the proximity of the saturation line necessitates considering curved domains (cf. Fig. 2, 3, 4, 5) which can be handled as in [8, 9] by introducing a curvilinear coordinate system on the domain $\mathscr{E}$ or equivalently mapping the domain $\mathscr{E}$ onto a cannonical domain $\mathscr{S}$. The construction of univalent (one-to-one) maps $\bar{U}$: $\mathscr{S} \rightarrow \mathscr{E}$ was the subject of [8] and the interested reader is referred to that reference for details. Suffice to say, that for regions as considered in this investigation, the mappings used are univalent.

4. Development of the Algorithm

An algorithm was developed to interpolate (approximate) one property of superheated steam as a function of two others. In the examples, enthalpy of superheated steam is calculated as a function of the pressure and temperature. Two different domains $\mathscr{E}$ are considered. The first, consists of the temperature range $635.8 \leqslant T \leqslant 700°\text{F}$ and pressure range $2000 \leqslant P \leqslant 2500$ psia. Table I lists the given data. As one can see, this is a very narrow range, but this region was used to establish a benchmark problem on which different routines can be tested and their features objectively compared. The domain $\mathscr{E}$ for the second

TABLE I

Portion of Tabulated Data: Enthalpy for Various Values of Pressure and Temperature

T \ P	2000	2100	2200	2300	2400	2500
635.8	1136.7*	√	√	√	√	√
640.0	1147.1*	√	√	√	√	√
642.7	1152.7	1128.9*	√	√	√	√
649.4	1166.6	1146.1	1120.5*	√	√	√
650.0	1167.8*	1147.6	1122.4	√	√	√
655.9	1178.2	1159.9	1138.0	1111.8*	√	√
660.0	1185.5*	1168.4	1148.9	1125.5	√	√
662.1	1188.8	1172.2	1153.2	1130.8	1102.3*	√
668.1	1198.3	1182.9	1165.7	1146.1	1122.7	1090.7*
670.0	1201.3*	1186.3	1169.6	1150.9	1129.1	1098.4*
680.0	1215.6*	1202.2	1187.6	1171.5	1153.6	1132.0*
690.0	1228.7*	1216.6	1203.6	1189.5	1174.0	1156.9*
700.0	1240.9*	1229.9*	1218.1*	1205.5*	1191.8*	1177.0*

* These points used to determine the enthalpy along the boundary of the transfinite element $\mathscr{E}$.

example consists of a much larger portion of the steam tables consisting of the temperature range $80 \leqslant T \leqslant 700°\text{F}$ and pressure range $0.5 \leqslant P \leqslant 2500$ psia.

Wide ranges of pressure and temperature can also be handled by decomposing a given region into various transfinite elements, each element being handled as described below. The smoothness of the resulting piecewise blended surface approximating enthalpy of superheated steam is governed by the smoothness of the map $\bar{U}$ and the smoothness of the blending functions in (4).

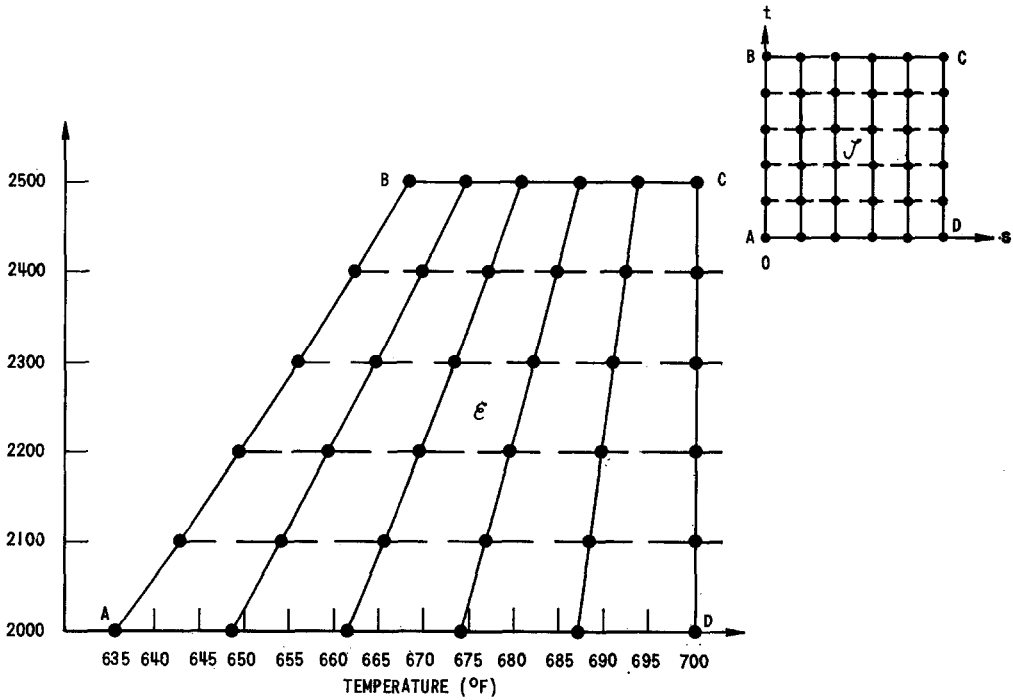

FIG. 3. A univalent mapping $\bar{U}$ induces a curvilinear coordinate system on the domain $\mathscr{E}$ in temperature–pressure plane. Curve AB is the saturation line.

The enthalpy of the superheated steam, when plotted in the temperature–pressure–enthalpy space, represents a surface as shown on Figs. 2 and 5. The approximation problem consists of choosing temperature and pressure independently and then determining enthalpy. By projecting this surface to the temperature–pressure plane one obtains a curvilinear domain as shown for example in Fig. 3.

Let us briefly develop some of the details of the transfinite interpolation scheme as given in [8] applied to the region in Fig. 3. Let $\mathscr{E}$ be a closed bounded simply connected region in the T, P-plane, whose boundary $ABCD$, is subdivided into

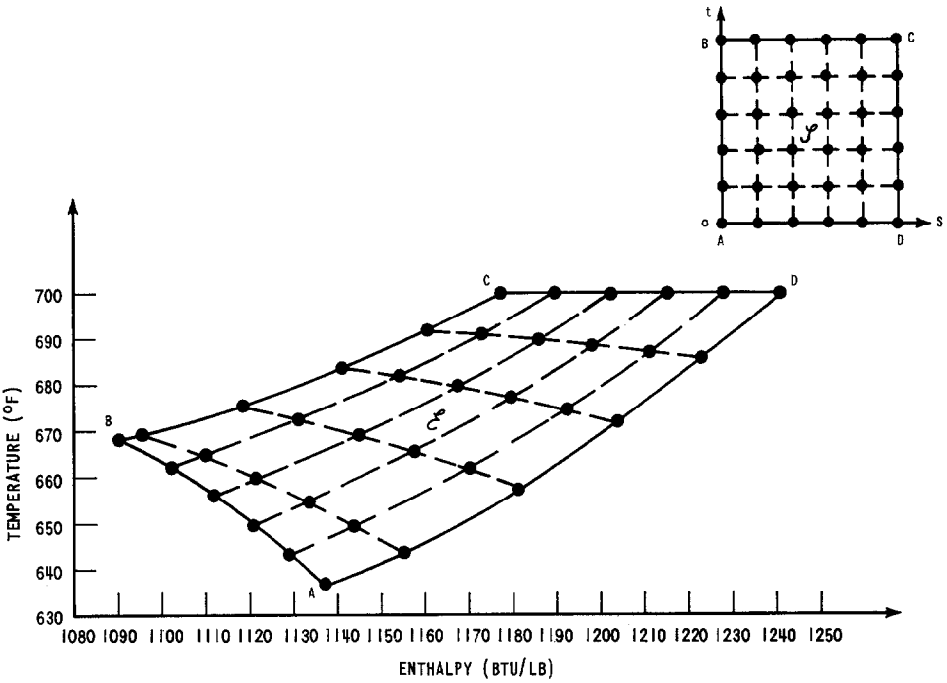

FIG. 4. Domain $\mathscr{E}$ plotted in enthalpy–temperature plane. Curve AB is saturation line.

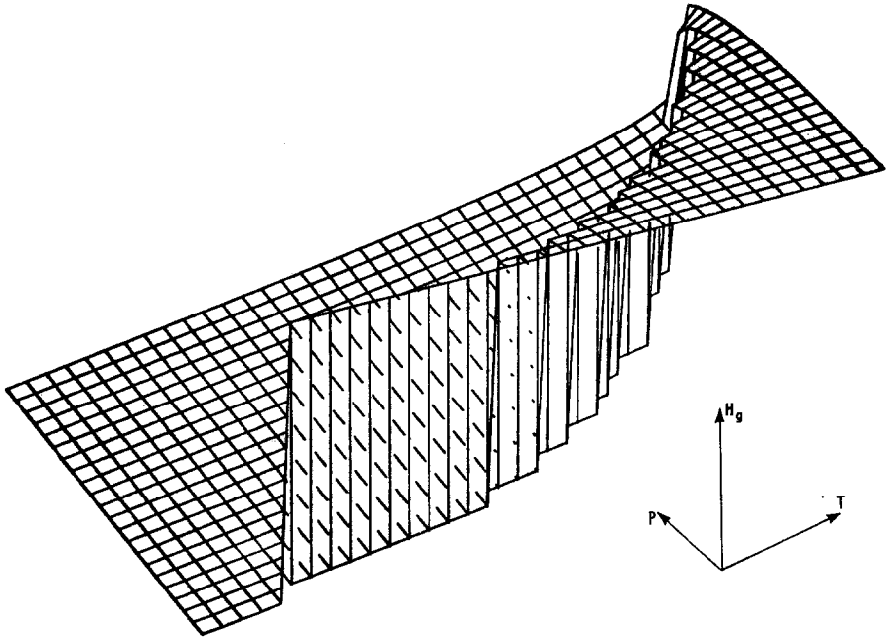

FIG. 5. Enthalpy of superheated steam versus temperature and pressure (wide range).

four curve segments AB, BC, CD, DA. Let $\overline{U}(s, t)$ be a univalent mapping of the unit square $\mathscr{S}$: $[0, 1] \times [0, 1]$ in the s, t-plane onto the region $\mathscr{E}$ in the T, P-plane

$$\overline{U}: \begin{pmatrix} s \\ t \end{pmatrix} \to \begin{pmatrix} T(s, t) \\ P(s, t) \end{pmatrix}. \tag{10}$$

Since $\overline{U}$ is univalent and onto, the boundary of $\mathscr{S}$ maps onto the boundary of $\mathscr{E}$ and the mapping is invertible, that is, the map $\overline{U}: \mathscr{S} \to \mathscr{E}$ provides a unique correspondence between a point $(s, t) \in \mathscr{S}$ and its image $(T, P) \in \mathscr{E}$. Therefore, a function $F(s, t)$ defined for $(s, t) \in \mathscr{S}$ will be transformed by $\overline{U}$, treating $\overline{U}$ as a change of variables, and we obtain $F^*(T, P) \equiv F(s(T, P), t(T, P))$ defined for all $(T, P) \in \mathscr{E}$. If $\mathscr{P}_s \oplus \mathscr{P}_t[F]$ is the (m, n)-*degree transfinite bivariate Lagrange interpolant* [9] it interpolates F along the lines $s = s_i$, $0 \leqslant i \leqslant m$ and $t = t_j$, $0 \leqslant j \leqslant n$. If $\overline{U}: \mathscr{S} \to \mathscr{E}$ is any univalent map of $\mathscr{S}$ onto $\mathscr{E}$ then as in [4, p. 892] the interpolant

$$\mathscr{P}_s \oplus \mathscr{P}_t[F]^* (T(s, t), \mathscr{P}(s, t)) \equiv \mathscr{P}_s \oplus \mathscr{P}_t[F](s, t) \tag{11}$$

and the domain $\mathscr{E}$ are defined to be a *transfinite element*.

Since the function F^* is defined for all $(T, P) \in \mathscr{E}$ by relation $F^*(T, P) = F(s, t)$, then $(\mathscr{P}_s \oplus \mathscr{P}_t[F])^*$ interpolates F^* along the two sets of curves in $\mathscr{E}$ which are the images under $\overline{U}$ of the family of lines $s = s_i$ and $t = t_j$ in $\mathscr{S}$, i.e.

$$\begin{aligned} (\mathscr{P}_s \oplus \mathscr{P}_t[F])^* (T(s_i, t), P(s_i, t)) &= F^*(T(s_i, t), P(s_i, t)) \qquad i = 0, 1, \ldots, m \\ (\mathscr{P}_s \oplus \mathscr{P}_t[F])^* (T(s, t_j), P(s, t_j)) &= F^*(T(s, t_j), P(s, t_j)) \qquad j = 0, 1, \ldots, n. \end{aligned} \tag{12}$$

During the course of this work linear splines as well as cubic splines were used to approximate $T(s_i, t)$, $P(s_i, t)$ etc. To minimize the number of nodes the node search program [6] was used.

The interpolation (approximation) problem described here is an "inverse problem." The point $P(s, t)$, $T(s, t)$ in the domain $\mathscr{E}$ is known, and in order to use the bivariate blended interpolant $\mathscr{P}_s \oplus \mathscr{P}_t[F]$, one must determine corresponding coordinates $(s, t) \in \mathscr{S}$. Once the appropriate (s, t) are known, then by the use of (11) one can determine $\mathscr{P}_s \oplus \mathscr{P}_t[F]^* (T(s, t), P(s, t))$. In fact, it is easy to see that F^* can be any thermodynamic property, e.g., enthalpy, entropy, or specific volume.

To test the described algorithm a digital computer program was written. In the first example the enthalpy of the superheated steam as a function of the temperature and pressure is approximated using transfinite elements. The region (cf. Fig. 3) considered is bounded by the saturation line and by the temperature of 700°F, while the pressure assumes values between 2000 and 2500 psia. The boundary $ABCD$ of the region $\mathscr{E}$ is divided into 4 curvilinear segments AB, BC, CD, DA.

In this particular case, three boundary curves are straight lines while the remaining one, AB is the curved saturation line.

The next step in the development of the algorithm is the parametrization of the boundary segments as shown on Table II. By using linear blending functions

TABLE II

Parametrization of Boundary Segments ($T(0, t)$ is a linear spline with 6 nodes)

Parameters s/t		Pressure	Temperature	Segment
s Variable	$t = 0$	$P(s, 0) = 2000$	$T(s, 0) = 700s + (1 - s)635.8$	AD
	$t = 1$	$P(s, 1) = 2500$	$T(s, 1) = 700s + (1 - s)668.1$	BC
$s = 0$	t	$P(0, t) = 2500t + (1 - t)*2000$	$T(0, t) = T_{\text{saturation}}$	AB
$s = 1$	Variable	$P(1, t) = 2500t + (1 - t)*2000$	$T(1, t) = 700$	CD

in accordance with (3), $\phi_0(s) = 1 - s$; $\phi_1(s) = s$; $\psi_0(t) = 1 - t$; $\psi_1(t) = t$; temperature and pressure can be written in the form of the Eq. (6) as

$$P(s, t) = (1 - s)\,P(0, t) + sP(1, t) + (1 - t)\,P(s, 0) + tP(s, 1) - [(1 - s)(1 - t)\,P(0, 0) + (1 - s)\,tP(0, 1) + s(1 - t)\,P(1, 0) + stP(1, 1)] \tag{13}$$

$$T(s, t) = (1 - s)\,T(0, t) + sT(1, t) + (1 - t)\,T(s, 0) + tT(s, 1) - [(1 - s)(1 - t)\,T(0, 0) + (1 - s)\,tT(0, 1) + s(1 - t)\,T(1, 0) + stT(1, 1)]. \tag{14}$$

By substituting expressions from Table II into Eqs. (13) and (14) and solving for s and t, respectively, one obtains

$$t = \frac{P(0, t) - P(0, 0)}{P(1, 1) - P(0, 0)} \tag{15}$$

$$s = \frac{T(s, t) - T(0, t)}{T(1, t) - T(0, t)}, \tag{16}$$

where $P(0, t)$ and $T(s, t)$ are known, given values, of the pressure and temperature. This closed form of the expressions (15) and (16) was obtained because of the special form of the domain $\mathscr{E}$ and because of the linear parametrization of the temperature and pressure along the three straight line boundary segments. In general, a system of nonlinear equations (13) and (14) has to be solved iteratively for s and t. This was the case when the domain $\mathscr{E}$ was plotted in the temperature–enthalpy plane and transfinite interpolation used to determine pressure (cf. Fig. 4).

When s and t are known, they are used in the expression for the enthalpy. Again, linear blending functions are used:

$$H(s,t) = (1-s)\,H(0,t) + sH(1,t) + (1-t)\,H(s,0) + tH(s,1) - [(1-s)(1-t)\,H(0,0) + (1-s)\,tH(0,1) + s(1-t)\,H(1,0) + stH(1,1)]. \tag{17}$$

Here functions $H(0,t)$, $H(1,t)$, $H(s,0)$ and $H(s,1)$ are the enthalpies corresponding to the parametrization of the temperature and pressure. Namely, $H(0,t)$ is the enthalpy at the saturation, $H(1,t)$ is the enthalpy vs. pressure at 700°F, $H(s,0)$ is the enthalpy vs. temperature at 2000 psia and $H(s,1)$ is the enthalpy vs. temperature at 2500 psia. The enthalpy data on the boundary of $\mathscr{E}$ can be approximated and stored as linear splines, cubic splines, etc., or supplied as a dense set of points calibrated in accordance with the anticipated values that s and t will be allowed to achieve in any table look-up.

From this development, it is obvious that the entropy or specific volume of superheated steam can be readily written in the form of the Eq. (17). Only four additional parametrized boundary curves are needed for each new property.

In summary, the algorithm can be outlined.

1. Determine the desired range of temperatures and pressures (domain $\mathscr{E}$),
2. Divide the boundary of the domain $\mathscr{E}$ into four curvilinear segments,
3. Parametrize the boundary, that is, determine independent variables (e.g. T and P) as a function of s, t, along each boundary segment,
4. Solve Eqs. (13) and (14) for s and t,
5. Calculate dependent variable (say enthalpy) by means of linear blending functions (17).

5. Examples

For the region in Fig. 3 the enthalpy was approximated using (17) where 6 data points were used along the curved saturation line AB, and 5, 6, and 8 data points along the straight boundary segments BC, CD, and AD, respectively. That is, from a total of 21 values (designated by an asterisk in Table I) of (P, T, H), the enthalpy H as a function of pressure and temperature is determined by (17).

To test the accuracy of approximation, the exact values from the steam tables were used for comparison. Specifically, at $T = 680$°F and $P = 2300$ psia the table value of enthalpy is 1171.5 BTU/lb. The interpolated value using (17) was 1171.312 for a relative error of 0.0135%. The relative error appears to range over $\mathscr{E}$ from 0.0032% to 0.0963% for this domain.

The tensor product bilinear interpolant (cf. (5)) for the patch determined by 4 data points $T = 670°F$, $690°F$ and $P = 2200$ psia and 2400 psia yield a relative error of 0.16% which is slightly larger than the corresponding relative error for the blended interpolant. Note however that to construct such a piecewise tensor product interpolant would require utilizing roughly two and one-half times the data as with the blended interpolant, that is the entire 53 values of (T, P, H) in Table I.

In the second example a much wider range ($80 \leqslant T \leqslant 700°F$ and $0.5 \leqslant P \leqslant 2500$ psia) was considered and enthalpy of superheated steam was approximated by using (17). In this case cubic splines were used to approximate curves $T(0, t)$, $H(0, t)$, $H(1, t)$, while $H(s, 0)$ and $H(s, 1)$ were approximated by linear splines. To make this approach more attractive to the prospective users, minimal number of nodes for a given pointwise error was required for the spline approximation. For this purpose the node search program described in [6] was used. Nine, eight, and eight nodes were used to determine the cubic splines $T(0, t)$, $H(0, t)$, and $H(1, t)$ respectively.

The accuracy of this approximation was tested and the results were compared to the ASME 1967 Steam Tables. Specifically, at $T = 680°F$ and $P = 2300$ psia the table value of superheated steam enthalpy is 1171.5 BTU/lb. The interpolated value using (17), and combination of cubic and linear splines as described above was 1163.915 BTU/lb for a relative error of 0.65%.

We note that the cubic spline approximation of the $T(0, t)$, $H(0, t)$ and $H(1, t)$ was accurate to approximately 0.05%. A better curve fit would yield more accurate results, as described in [9]. By using boundary values for $T(0, t)$, $H(0, t)$, $H(1, t)$, etc., from the tables (no approximation) the error obtained was 0.12%.

To construct the linear blended approximant to enthalpy using spline approximations to boundary data as described above involves 11 (T, P) data and 16 (H) data at the boundary points. In comparison, a tensor product bilinear interpolant based on, for example, increments of 10°F in temperature and 100 psia in pressure for the same domain $\mathscr{E}$ would require approximately 1100 values of (T, P, H) and would require *ad hoc* strategies near the saturation line.

To make these techniques more versatile two additional routines were written. In the first routine any one thermodynamic parameter can be calculated from the other two, while in the second, for given pair of the independent variables three dependent variables can be calculated. Both approaches require minimal additional data in comparison with the basic case.

When written in FORTRAN in the form of a function routine, this algorithm required approximately 600 words of storage (including 3 cubic spline approximations).

7. Conclusion

Blending function techniques were used to construct a transfinite domain transformation and to interpolate (approximate) steam and water properties. The method consists of mapping the unit square onto the region of interest in the temperature–pressure plane, for example, constructing the domain transformation and using linear blending functions to calculate the dependent variable. For specific regions the shape of the domain can be used to develop simple relationships which circumvent the necessity of solving nonlinear systems, enhancing speed of the calculations. Storage requirements for the technique are considerably less than for the existing tensor product schemes of comparable accuracy.

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